

Charged lepton correction to tribimaximal lepton mixing and its implications to neutrino phenomenology

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Abstract

The recent results from Daya Bay and RENO reactor neutrino experiments have firmly established that the smallest reactor mixing angle θ_{13} is non-vanishing at the 5σ level, with a relatively large value, i.e., $\theta_{13} \approx 9^\circ$. Using the fact that the neutrino mixing matrix can be represented as $V_{\text{PMNS}} = U_l^\dagger U_\nu P_\nu$, where U_l and U_ν result from the diagonalization of the charged lepton and neutrino mass matrices and P_ν is a diagonal matrix containing the Majorana phases and assuming the tri-bimaximal form for U_ν , we investigate the possibility of accounting for the large reactor mixing angle due to the corrections of the charged lepton mixing matrix. The form of U_l is assumed to be that of CKM mixing matrix of the quark sector. We find that with this modification it is possible to accommodate the large observed reactor mixing angle θ_{13} . We also study the implications of such corrections on the other phenomenological observables.

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I. INTRODUCTION

The results from various neutrino oscillation experiments firmly established the fact that neutrinos have a tiny but finite nonzero mass. Thus, analogous to the mixing in the down-quark sector, the three flavor eigenstates of neutrinos (ν_e, ν_μ, ν_τ) are related to the corresponding mass eigenstates (ν_1, ν_2, ν_3) by the unitary transformation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (1)$$

where V is the 3×3 unitary matrix known as PMNS matrix [1], which contains three mixing angles and three CP violating phases (one Dirac type and two Majorana type). In the standard parametrization [2], V_{PMNS} is expressed in terms of the solar, atmospheric and reactor mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and three CP-violating phases $\delta_{\text{CP}}, \rho, \sigma$ as

$$V_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\text{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\text{CP}}} & c_{13}c_{23} \end{pmatrix} P_\nu \equiv U_{\text{PMNS}} P_\nu, \quad (2)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ and $P_\nu \equiv \{e^{i\rho}, e^{i\sigma}, 1\}$ is a diagonal matrix with CP violating Majorana phases ρ and σ . The neutrino oscillation data accumulated over many years allow us to determine the solar and atmospheric neutrino oscillation parameters with very high precision. Recently, the value of the smallest mixing angle θ_{13} has been measured by the Daya Bay [3] and RENO Collaborations [4] with the best fit (1σ) result as

$$\begin{aligned} \sin^2 2\theta_{13} &= 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}), & \text{Daya Bay} \\ \sin^2 2\theta_{13} &= 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst}). & \text{RENO} \end{aligned} \quad (3)$$

which is equivalent to $\theta_{13} \simeq 8.8^\circ \pm 0.8^\circ$. This is 5.2σ evidence of nonzero value of θ_{13} which confirms the previous measurements of T2K [5], MINOS [6] and Double Chooz [7] experiments. The global analysis of the recent results of various neutrino oscillation experiments has been performed by several groups [8–10], and the parameters which are used in this analysis are taken from Ref. [10], are presented in Table-1.

The observation of this not so small reactor mixing angle θ_{13} has ignited a lot of interest to understand the mixing pattern in the lepton sector [11]. It opens promising perspectives

TABLE I: The global fit values of the mixing parameters taken from [10].

Mixing Parameters	1 σ value	3 σ Range
$\sin^2 \theta_{12}$	$0.302^{+0.013}_{-0.012}$	$0.267 \rightarrow 0.344$
$\theta_{12}/^\circ$	$33.36^{+0.81}_{-0.78}$	$31.09 \rightarrow 35.89$
$\sin^2 \theta_{23}$	$0.413^{+0.037}_{-0.025}$	$0.342 \rightarrow 0.667$
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5}$	$35.8 \rightarrow 54.8$
$\sin^2 \theta_{13}$	$0.0227^{+0.0023}_{-0.0024}$	$0.0156 \rightarrow 0.0299$
$\theta_{13}/^\circ$	$8.66^{+0.44}_{-0.46}$	$7.19 \rightarrow 9.96$
$\delta_{\text{CP}}/^\circ$	300^{+66}_{-138}	$0 \rightarrow 360$
$\Delta m_{21}^2/10^{-5}\text{eV}^2$	$7.5^{+0.18}_{-0.19}$	$7.00 \rightarrow 8.09$
$\Delta m_{31}^2/10^{-3}\text{eV}^2(\text{NH})$	$2.473^{+0.07}_{-0.067}$	$2.276 \rightarrow 2.695$
$\Delta m_{32}^2/10^{-3}\text{eV}^2(\text{IH})$	$-2.427^{+0.042}_{-0.065}$	$-2.649 \rightarrow -2.247$

for the observation of CP violation in the lepton sector. The precise determination of θ_{13} in addition to providing a complete picture of neutrino mixing pattern, could be a signal of underlying physics responsible for lepton mixing and for the physics beyond standard model. It has been shown that if one includes some perturbative corrections to the leading order neutrino mixing patterns, such as bi-maximal (BM) [12], tri-bimaximal (TBM) [13] and democratic (DC) [14], it is possible to explain the observed neutrino mixing angles [15]. However, it should be noted that among these leading order mixing patterns i.e., BM, TBM and DC, the tri-bimaximal pattern, whose explicit form as given below

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}, \quad (4)$$

is particularly very interesting. It corresponds to the three mixing angles of the standard parametrization as $\theta_{12} = \arctan(1/\sqrt{2}) \simeq 35.3^\circ$, $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$. Clearly, to accommodate the large value of θ_{13} , one has to consider possible perturbations to the TBM mixing matrix. In this paper we would like to study the possible corrections arising from the charged lepton sector. The essential features of our analysis are as follows. We assume the charged lepton mixing matrix to be of the same form as the CKM quark mixing matrix and the neutrino mixing matrix to be of the tri-bimaximal form. Furthermore, we use

the Wolfenstein-like parametrization for the charged lepton mixing matrix and study its implications on various phenomenological observables. It should be noted that there have been several attempts made recently to understand the nonzero θ_{13} due to charged lepton correction [16] and in the past also corrections to the leptonic mixing matrix due to charged leptons were considered in Ref. [17].

The paper has been organized as follows. The methodology of our analysis is presented in Section-II and the Results and Conclusion are discussed in Section-III.

II. METHODOLOGY

It is well known that the leptonic mixing matrix arises from the overlapping of the matrices that diagonalize charged lepton and neutrino mass matrices

$$U_{\text{PMNS}} = U_l^\dagger U_\nu . \quad (5)$$

For the study of leptonic mixing it is generally assumed that the charged lepton mixing matrix as an identity matrix and the neutrino mixing matrix U_ν has a specific form dictated by a symmetry which fixes the values of the three mixing angles in U_ν . The small deviations of the mixing angles from those measured in the PMNS matrix, are considered, in general, as perturbative corrections arising from symmetry breaking effects. A variety symmetry forms of U_ν have been explored in the literature e.g., BM/TBM/DC and so on. In this work we will consider the situation wherein the neutrino mixing matrix is described by the TBM matrix, i.e.,

$$U_\nu = U_{\text{TBM}} , \quad (6)$$

and that the mixing angles induced by the charged leptons can be considered as corrections. Furthermore, we will neglect possible corrections to U_{TBM} from higher dimensional operators and from renormalization group effects. In this approximation we will derive formulae which allow us to include corrections to neutrino mixing angles and to constrain the CP violating phase (δ_{CP}) conveniently.

In our study, we use a simple *ansatz* for the charged lepton mixing matrix U_l , i.e., we assume that U_l has the same structure as the CKM matrix connecting the weak eigenstates of the down type quarks to the corresponding mass eigenstates. This approximation is quite reasonable as we know that the CKM matrix is almost diagonal with the off diagonal

elements strongly suppressed by the small expansion parameter $\lambda = \sin \theta_C$ (θ_C , being the Cabibbo angle). Hence, such an assumption can naturally provide the small perturbations to the tri-bimaximal mixing pattern for neutrino mixing matrix. Furthermore, as discussed in Ref. [18], this approximation is quite acceptable as the mass spectrum of charged leptons exhibits similar hierarchical structure as the down type quarks, i.e., $(m_e, m_\mu) \approx (\lambda^5, \lambda^2)m_\tau$ and $(m_d, m_s) \approx (\lambda^4, \lambda^2)m_b$. This may imply that the charged lepton mixing matrix has a structure similar to the down type quark mixing and is governed by the CKM matrix.

To illustrate the things more explicitly, let us recall the values of the quark mixing angles in the standard PDG parametrization for the CKM matrix within 1σ range as [19]

$$\theta_{13}^q = 0.20^\circ \pm 0.01^\circ, \quad \theta_{23}^q = 2.35^\circ \pm 0.07^\circ, \quad \theta_{12}^q \equiv \theta_C = 13.02^\circ \pm 0.04^\circ. \quad (7)$$

However, the leptonic sector is described by two large mixing angles θ_{23}^l and θ_{12}^l and the third mixing angle θ_{13}^l , was expected to be very small. Recently, the third mixing angle θ_{13}^l has been measured by T2K, Double CHOOZ, Daya Bay and RENO Collaborations yielding the following mixing patterns in the lepton sector:

$$\theta_{13}^l = 8.8^\circ \pm 1.0^\circ, \quad \theta_{23}^l = 40.4^\circ \pm 1.0^\circ, \quad \theta_{12}^l = 34.0^\circ \pm 1.1^\circ. \quad (8)$$

The different nature of the quark and lepton mixing angles can be inter-related in terms of the quark lepton complementarity (QLC) relations [20], as

$$\theta_{12}^q + \theta_{12}^l \simeq 45^\circ, \quad \theta_{23}^q + \theta_{23}^l \simeq 45^\circ. \quad (9)$$

The QLC relations indicate that it could be possible to have a quark-lepton symmetry based on some flavor symmetry. The experimental result of this not-so-small reactor mixing angle θ_{13}^l has triggered a lot of interest in the theoretical community. Given the rather precise measurement of θ_{13}^l , one may wonder whether θ_{13}^l numerically agrees well with the QLC relation, i.e.,

$$\theta_{13}^l = \frac{\theta_C}{\sqrt{2}} \approx 9.2^\circ. \quad (10)$$

In particular, it is quite interesting to see whether this specific connection to θ_C can be a consequence of some underlying symmetry, which may provide a clue to the nature of quark-lepton physics beyond the standard model.

Starting from the fact that the mixing matrix of the up type quark sector can be almost diagonal and so the CKM matrix is mainly generated from the down type quark mixing

matrix, we assume that the mixing matrix of the charged lepton sector is basically of the same form as that of down type quark sector. Consequently the lepton mixing matrix appears as the product of CKM like matrix (induced by charged lepton sector) and the TBM pattern matrix induced from the neutrino sector. As discussed before, in the limit of diagonal charged lepton mass matrix i.e., $U_l = \mathbf{1}$, and $U_\nu = U_{\text{TBM}}$, which gives the mixing angles at the leading order as

$$\theta_{12}^{l0} = \arctan(1/\sqrt{2}) \simeq 35.3^\circ, \quad \theta_{13}^{l0} = 0^\circ, \quad \text{and} \quad \theta_{23}^{l0} = 45^\circ, \quad (11)$$

deviate significantly from their measured values as

$$|\theta_{12}^l - \theta_{12}^{l0}| \simeq 2^\circ, \quad \theta_{13}^l - \theta_{13}^{l0} \approx 9^\circ \quad \text{and} \quad |\theta_{23}^l - \theta_{23}^{l0}| \simeq 5^\circ. \quad (12)$$

These deviations are attributed to the corrections arising from the charged lepton sector. We assume the charged lepton mixing matrix to have the form as the CKM matrix in the standard parametrization, i.e.,

$$U_l = R_{23}U_{13}R_{12}, \quad (13)$$

where the matrices R_{23} , U_{13} and R_{12} are defined by

$$R_{12} = \begin{pmatrix} \cos \theta_{12}^l & \sin \theta_{12}^l & 0 \\ -\sin \theta_{12}^l & \cos \theta_{12}^l & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23}^l & \sin \theta_{23}^l \\ 0 & -\sin \theta_{23}^l & \cos \theta_{23}^l \end{pmatrix}$$

$$U_{13} = \begin{pmatrix} \cos \theta_{13}^l & 0 & \sin \theta_{13}^l e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin \theta_{13}^l e^{i\delta} & 0 & \cos \theta_{13}^l \end{pmatrix}. \quad (14)$$

Furthermore, as the mixing angle θ_{13} receives maximum deviation from the TBM pattern, we assume $\sin \theta_{13}^l = \sin \theta_C = \lambda$, where, λ is a small expansion parameter analogous to the expansion parameter of Wolfenstein parametrization of the CKM matrix. The other two angles are assumed to be of the form

$$\sin \theta_{23}^l = A\lambda^2, \quad \sin \theta_{12}^l = A\lambda^3, \quad (15)$$

where the parameter $A = \mathcal{O}(1)$. With these values, one can obtain the Wolfenstein-like

parametrization for U_l (upto order λ^3) as

$$U_l = \begin{pmatrix} 1 - \lambda^2/2 & A\lambda^3 & \lambda e^{-i\delta} \\ -A\lambda^3(1 + e^{i\delta}) & 1 & A\lambda^2 \\ -\lambda e^{i\delta} & -A\lambda^2 & 1 - \lambda^2/2 \end{pmatrix} \quad (16)$$

Thus, with the help of Eqs. (5), (6) and (16), one can schematically obtain the PMNS matrix up to order of λ^3 as

$$U_{\text{PMNS}} = U_{\text{TBM}} + \Delta U, \quad (17)$$

with

$$\Delta U = \begin{pmatrix} \frac{\lambda e^{-i\delta} - \lambda^2 + A\lambda^3(1 + e^{-i\delta})}{\sqrt{6}} & -\frac{\lambda e^{-i\delta} + \lambda^2/2 + A\lambda^3(1 + e^{-i\delta})}{\sqrt{3}} & -\frac{\lambda e^{-i\delta} - A\lambda^3(1 + e^{-i\delta})}{\sqrt{2}} \\ \frac{A\lambda^2(1 + 2\lambda)}{\sqrt{6}} & -\frac{A\lambda^2(1 - \lambda)}{\sqrt{3}} & -\frac{A\lambda^2}{\sqrt{2}} \\ \frac{2\lambda e^{i\delta} - A\lambda^2 + \lambda^2/2}{\sqrt{6}} & \frac{\lambda e^{i\delta} + A\lambda^2 - \lambda^2/2}{\sqrt{3}} & -\frac{A\lambda^2 + \lambda^2/2}{\sqrt{2}} \end{pmatrix} P_\nu + \mathcal{O}(\lambda^4), \quad (18)$$

which allows one to obtain the elements of the PMNS matrix as

$$\begin{aligned} |U_{e1}| &= \sqrt{\frac{2}{3}} \left[1 + \frac{1}{2}\lambda \cos \delta - \frac{1}{8}\lambda^2(3 + \cos^2 \delta) + \frac{1}{16}\lambda^3(8A(1 + \cos \delta) - \cos \delta \sin^2 \delta) \right], \\ |U_{e2}| &= \frac{1}{\sqrt{3}} \left[1 - \lambda \cos \delta - \frac{1}{2}\lambda^2 \cos^2 \delta - \frac{1}{2}\lambda^3(2A(1 + \cos \delta) - \cos \delta \sin^2 \delta) \right], \\ |U_{e3}| &= \frac{\lambda}{\sqrt{2}} [1 - A\lambda^2(1 + \cos \delta)], \\ U_{\mu 1} &= -\frac{1}{\sqrt{6}} [1 - A\lambda^2 - 2A\lambda^3], \\ U_{\mu 2} &= \frac{1}{\sqrt{3}} [1 - A\lambda^2 + A\lambda^3], \\ |U_{\mu 3}| &= \frac{1}{\sqrt{2}} (1 + A\lambda^2), \\ U_{\tau 1} &= -\frac{1}{\sqrt{6}} \left(1 - 2\lambda e^{i\delta} + \frac{1}{2}\lambda^2(2A - 1) \right), \\ U_{\tau 2} &= \frac{1}{\sqrt{3}} \left(1 + \lambda e^{i\delta} + \frac{1}{2}\lambda^2(2A - 1) \right), \\ |U_{\tau 3}| &= \frac{1}{\sqrt{2}} \left(1 - \frac{1}{2}\lambda^2 - A\lambda^2 \right). \end{aligned} \quad (19)$$

From Eq. (2), one can express the neutrino mixing parameters in terms of the PMNS mixing matrix elements as

$$\begin{aligned} \sin^2 \theta_{12} &= \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}, & \sin^2 \theta_{23} &= \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2}, \\ \sin \theta_{13} &= |U_{e3}|. \end{aligned} \quad (20)$$

Thus, from Eqs. (19) and (20), one can obtain the solar neutrino mixing angle θ_{12} , up to order λ^3 , as

$$\sin^2 \theta_{12} \simeq \frac{1}{3} \left(1 - 2\lambda \cos \delta + \frac{\lambda^2}{2} - \lambda^3 [2A(1 + \cos \delta) + \cos^3 \delta] \right), \quad (21)$$

Clearly, when $\cos \delta$ approaches zero we observe a tiny deviation from $\sin^2 \theta_{12} = 1/3$. Following similar approach, one can obtain the atmospheric neutrino mixing angle θ_{23} as

$$\sin^2 \theta_{23} \simeq \frac{1}{2} \left(1 + \frac{\lambda^2}{2}(1 + 4A) \right), \quad (22)$$

which also shows a small deviation from the maximal mixing pattern i.e., $\sin^2 \theta_{23} = 1/2$. The reactor mixing angle θ_{13} can be obtained as

$$\sin \theta_{13} = \frac{\lambda}{\sqrt{2}} (1 - A\lambda^2(1 + \cos \delta)). \quad (23)$$

Thus, we have a non-vanishing large θ_{13} . This in turn implies that it could be possible to observe CP violation in the lepton sector analogous to the quark sector, which could be detected through long base-line neutrino oscillation experiments. The Jarlskog invariant, which is a measure of CP violation, for the lepton sector has the expression

$$J_{\text{CP}}^\ell \equiv \text{Im}[U_{e1}U_{\mu 2}U_{\mu 1}^*U_{e2}^*] = -\frac{\lambda \sin \delta}{6} \left(1 - \frac{\lambda^2}{2} - A\lambda^2 \right) + \mathcal{O}(\lambda^4), \quad (24)$$

which is sensitive to the Dirac CP violating phase.

The Dirac CP phase δ_{CP} can be deduced by using the PMNS matrix elements and the neutrino mixing parameters as [18]

$$\delta_{\text{CP}} = -\arg \left(\frac{\frac{U_{e1}^*U_{e3}U_{\tau 1}U_{\tau 3}^*}{c_{12}c_{13}^2c_{23}s_{13}} + c_{12}c_{23}s_{13}}{s_{12}s_{23}} \right). \quad (25)$$

With Eqs. (4), (17) and (18), this yields the correlation between the two CP violating phases (Dirac type CP violating phase and the phase δ introduced in the charged lepton mixing)

$$\delta_{\text{CP}} = -\arctan \left[\frac{-\lambda(1 - (A + \frac{1}{2}\lambda^2)) \sin \delta}{\lambda \left[(A(1 - \lambda^2) - \frac{5}{2}\lambda^2) \cos \delta - (\frac{3}{2}\lambda + A\lambda^2) \right] + 6c_{12}^2c_{23}^2s_{13}^2c_{13}^2} \right]. \quad (26)$$

Three mass-dependent neutrino observables are probed in different types of experiments. The sum of absolute neutrino masses $\sum_i m_i$ is probed in cosmology, the kinetic electron neutrino mass in beta decay (M_β) is probed in direct search for neutrino masses, and the

effective mass (M_{ee}) is probed in neutrinoless double beta decay experiments with the decay rate for the process $\Gamma \propto M_{ee}$. In terms of the bare physical parameters m_i and $U_{\alpha i}$, the observables are given by [21]

$$\begin{aligned}\sum_i m_i &= m_1 + m_2 + m_3, \\ M_{ee} &= \sum_i U_{ei}^2 m_i, \\ M_\beta &= \sqrt{\sum_i |U_{ei}|^2 m_i^2}.\end{aligned}\tag{27}$$

The absolute values of neutrino masses are currently unknown. The Cosmic Microwave Background (CMB) data of the WMAP experiment, combined with the supernova data and data on galaxy clustering can be used to obtain an upper limit on the sum of neutrino masses [22]

$$\sum_i m_i \leq 0.68 \text{ eV} \quad 95\% \text{C.L.}\tag{28}$$

The Mainz [23] and Troitsk [24] experiments on the high precision measurement of the end-point beta decay spectrum of ${}^3\text{H}$, found

$$M_\beta < 2.3 \text{ eV} \quad 95\% \text{C.L.}\tag{29}$$

In our analysis we ignore the Majorana phases (ρ, σ) and consider the normal hierarchy scenario for the neutrino mass spectrum in which the neutrino masses m_2 and m_3 can be expressed in terms of the lightest neutrino mass m_1 as

$$m_2 = \sqrt{m_1^2 + \Delta m_{\text{sol}}}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{\text{sol}} + \Delta m_{\text{atm}}}.\tag{30}$$

III. RESULTS AND DISCUSSION

For numerical estimation we need to know the values of the three unknown parameters A , λ and δ . In this analysis we assume the small expansion parameter λ to have the same value as that of the quark sector [19]:

$$\lambda = 0.22535 \pm 0.00065.\tag{31}$$

Now with Eq. (22) and using the experimental value of $\sin^2 \theta_{23}$ as input parameter, we obtain the 1σ (3σ) range of A as

$$\begin{aligned} A &= (-2.4 \rightarrow -1.2) \quad (1\sigma) \\ &= (-3.4 \rightarrow 3.0), \quad (3\sigma) \end{aligned} \quad (32)$$

and we treat the CP violating phase δ as a free parameter, i.e., we allow it to vary in its entire range $0 \leq \delta \leq 2\pi$. Now varying these input parameters in their 3σ ranges, and using Eqs. (21) and (23), we present the variation of the solar and reactor mixing angles ($\sin^2 \theta_{12}$ and $\sin \theta_{13}$) with the CP violating phase δ in Fig-1. From the figure, it can be seen that in this formalism, it is possible to accommodate simultaneously the observed value of the reactor mixing angle θ_{13} and solar mixing angle θ_{12} . The correlation plots between the solar and atmospheric mixing angles with θ_{13} is shown in Figure-2. In Figure-3, we show the variation of the Jarlskog Invariant J_{CP} with δ and θ_{13} . From the figure it can be seen that it could be possible to have large CP violation $\mathcal{O}(10^{-2})$ in the lepton sector. The correlation between the Dirac CP violating phase δ_{CP} and the CP violating parameter δ of the charged lepton mixing matrix is shown in Fig-4. show a small deviation from the maximal mixing pattern i.e., $\sin^2 \theta_{23} = 1/2$ The variation of M_{ee} with the lightest neutrino mass m_1 (for Normal Hierarchy) and the variation of M_β with $\sum m_i$ (where the parameters are varied in their 1σ range) are shown in Fig-5. Thus, for m_1 below $\mathcal{O}(10^{-2})$ eV, one can get $M_{ee} \leq 1.2 \times 10^{-2}$ eV and $M_\beta \leq 1.4 \times 10^{-2}$ eV.

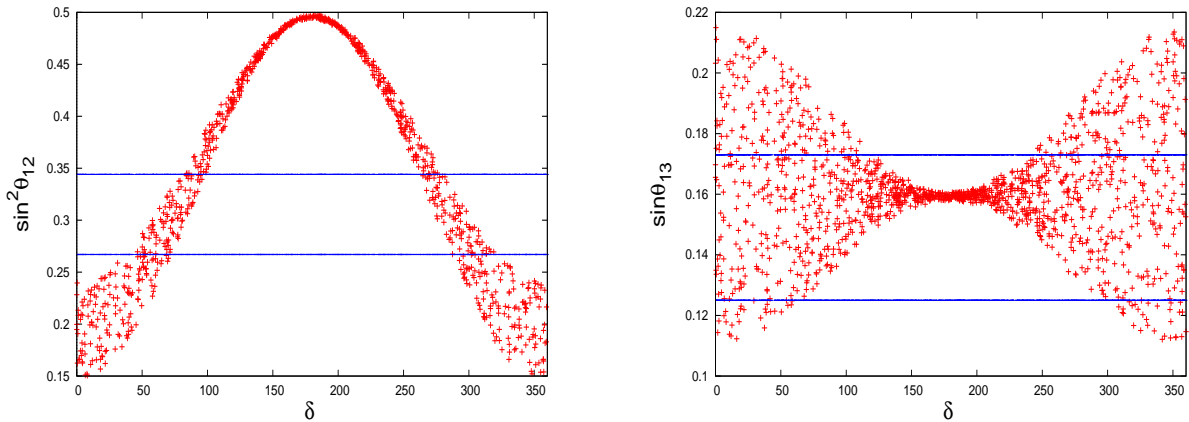


FIG. 1: Variation of $\sin^2 \theta_{12}$ with the CP violating phase δ (left panel) and $\sin \theta_{13}$ on the right panel. The horizontal lines (in both panels) represent the 3σ allowed range.

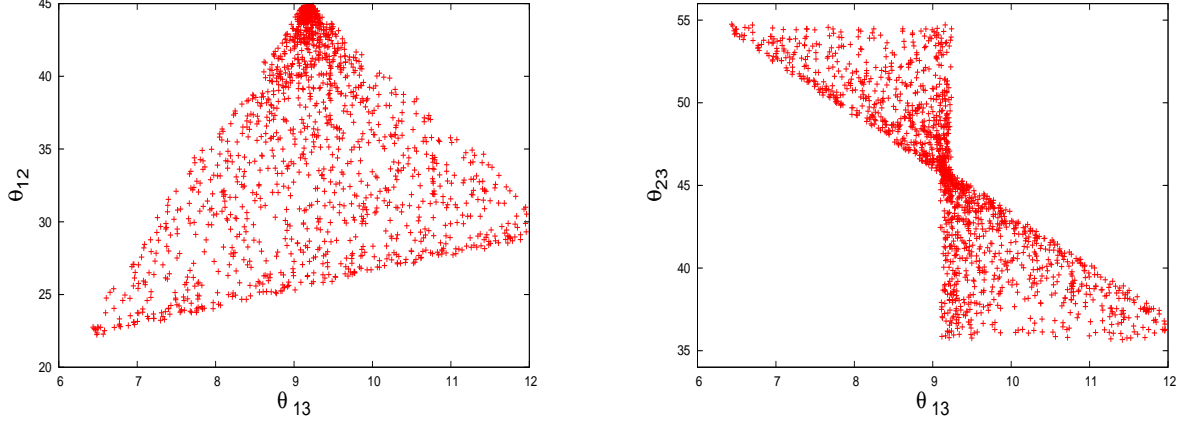


FIG. 2: Correlation plot between solar (left panel) and the atmospheric mixing angle (right panel) with θ_{13} .

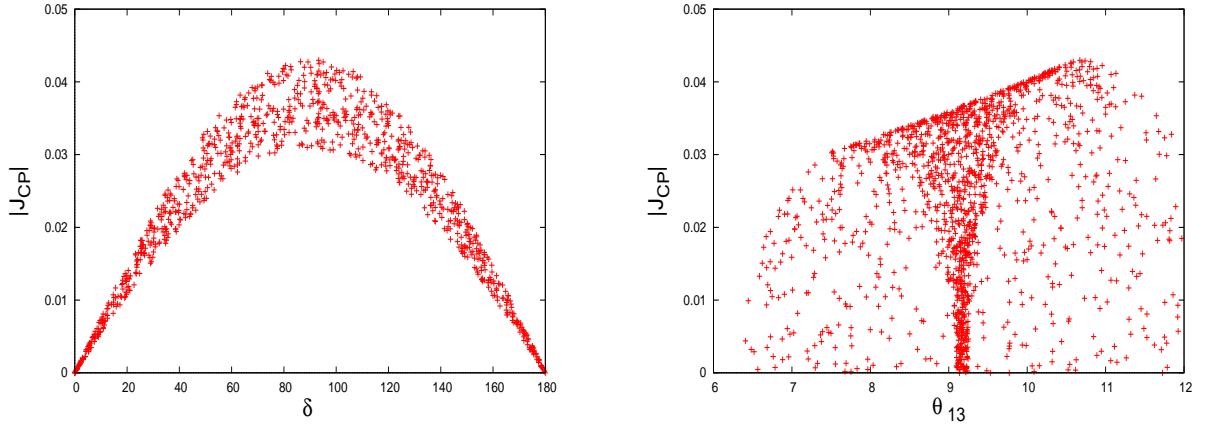


FIG. 3: Variation of J_{CP} with δ (left panel) and with θ_{13} (right panel).

To summarize, to accommodate the observed value of relatively large θ_{13} , we consider the corrections due to the charged lepton mixing matrix to the TBM pattern of neutrino mixing matrix. Based on the possible inter-relation between the charged lepton and the quark mixing structures we constructed the lepton mixing matrix to have the form of the CKM-like matrix (induced from the charged lepton sector) times the TBM matrix induced from the neutrino sector. Our result showed that in this formalism, it is possible to accommodate the observed reactor mixing angle θ_{13} along with the other mixing parameters within their experimental range. We have also found that sizable leptonic CP violation characterized by the Jarlskog invariant J_{CP} , i.e., $|J_{CP}| \leq 10^{-2}$ could be possible in this scenario. We have

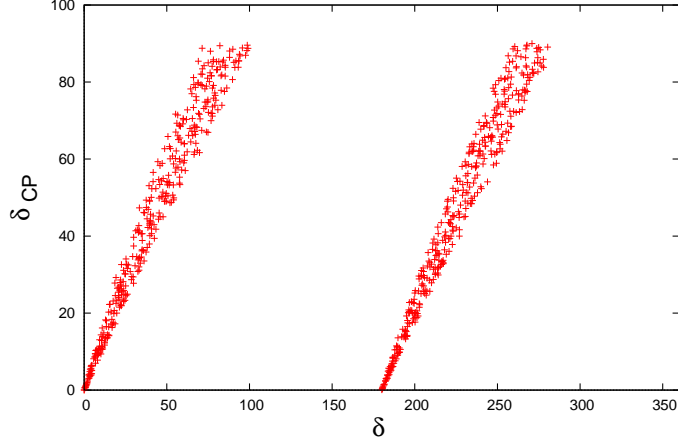


FIG. 4: The correlation plot between the Dirac CP violating phase δ_{CP} and δ .

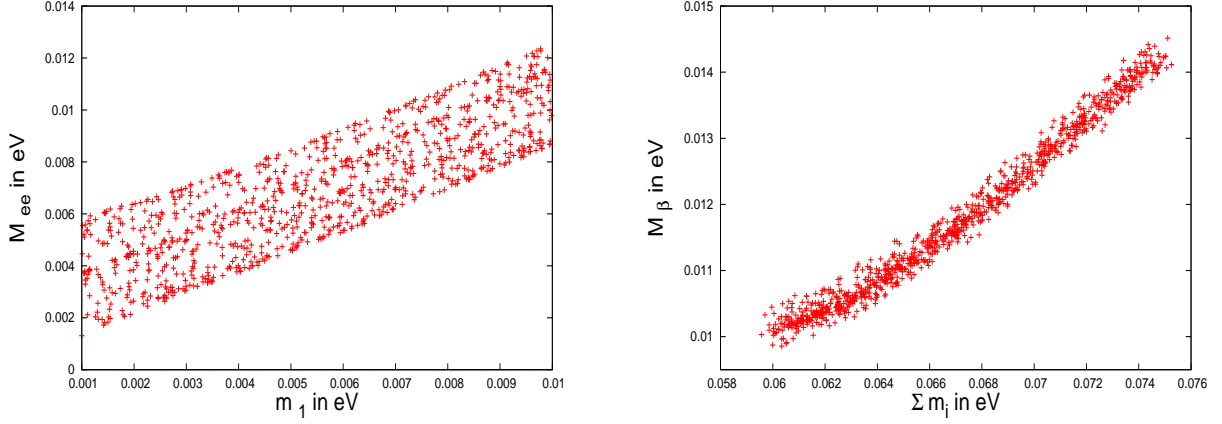


FIG. 5: Variation of M_{ee} with the lightest neutrino mass m_1 (left panel) and the variation of M_β with $\sum m_i$ (right panel).

also shown that the measured value of θ_{13} along with other mixing parameters can be used for constraining the value of the Dirac CP violating phase δ_{CP} . The upper limits on M_{ee} and M_β are found to be $\mathcal{O}(10^{-2})$, if the mass of the lightest neutrino $m_1 \leq 0.01$ eV. These predictions can be tested in the upcoming long base line neutrino experiments.

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